AIAA 81-1328R

Bulkheads in Airships

Donald E. Woodward*
Association of Balloon and Airship Constructors, Alexandria, Va.

Loading of the transverse frames of airships by pressure on the bulkheads is affected by pretensioning to give metacentric stability against gas surging. Known results for radial wire bulkheads are incomplete because they depend on the unknown pretensions. The necessary pretensions to limit the reduction in metacentric height to a specified value are derived for bulkheads with or without axial restraint. A numerical comparison is made of various types of bulkhead construction.

Nomenclature

a = wire section area
c = loading intensity $= 2\pi p/n$
E = wire elastic modulus
f = transverse load per unit length
h_0 = longitudinal metacentric height
h_1 = reduction in metacentric height
H = height of bulge
k = gas lift per unit volume
L = length of gas cell
n = number of radial wires
n' = number of frame joints
p = pressure of gas
P = total transverse load on wire
r = radial dimension
R = radius of gas cell
$\mathcal{R}$ = radius of curvature
s = wire tensile stress
S = linear tension (per unit length)
T = wire tension
T_0 = initial pretension value of T
V = bulge volume
$\maltese$ = gas cell volume
X = displacement of center of bulkhead
Y = normalized radius $= r/R$
z = axial dimension
Z = normalized axial dimension $= 6Tz/cR^3$
θ = angle between bulkhead tangent and transverse plane
ϕ = angle of pitch of airship

Introduction

LARGE airships have usually been divided by transverse bulkheads into several gas compartments. There are a number of reasons for this: 1) to provide safety against grounding as a result of accidental emptying of a single gas container; 2) to prevent gas surging which reduces metacentric stability; 3) to reduce the gas pressure head at the high end of the hull when the ship is inclined; and 4) to permit cross-bracing of the hull for transverse rigidity. The maximum size of the individual compartments is limited to a volume that will not result in unmanageable heaviness or out-of-trim moment if it is accidentally emptied.

A bulkhead consists of the flat fabric ends of adjacent gas cells and a supporting structure of transverse wires anchored to a rigid circular or polygonal transverse frame. These transverse wires are normally also the hull cross-bracing members, and as such have important roles to play in maintaining the circularity of the hull and in enabling it to conform to Saint Venant's principle.

When subjected to unbalanced pressures, the bulkheads exert inward forces on the frames to which they are anchored. If between a full cell and one that is completely deflated, these bulkhead forces may be very great and are typically the critical loads for which the frames must be designed. The force exerted on a vertical surface by a static pressure head, due to the lifting gas, is equal to the pressure at its centroid multiplied by its area. For a circular hull cross section, the centroidal pressure is kR and the total load πkR^3. The bulkhead bulges under pressure so that its edge meets the anchoring frame at an angle $\theta(R)$. If there is no center restraint, the longitudinal component of the force on the frame, per unit circumference, is $\frac{1}{2}kR^2$, and the radial component $\frac{1}{2}kR^2\cot\theta$. Since $\cot\theta$ is large for small θ, it is obviously best for the bulkhead to bulge as much as possible. However, the amount of bulging is limited by the initial tension in the bulkhead that is needed to prevent instability arising from gas surging.

When the airship is pitched (say, up by the bow) there is a difference in gas pressure on the two sides of the bulkhead equal to the unit lift times the height of the bottom of the bulkhead at the forward end of the cell above that at the after end. For a cylindrical cell, the pressure difference on the forward bulkhead will be $kL\sin\phi$ (Fig. 1). There will be an equal pressure across the aft bulkhead and both will bulge forward. The net result is that a volume V of gas surges forward a distance $L\cos\phi$ and causes an upsetting moment of $kVL\cos\phi$ that acts to increase ϕ. (Since ϕ is always small, $\cos\phi$ will be taken as 1 from here on.) V, and thus the instability moment, decreases when the bulkhead is restrained from bulging, so that the stability and frame-load requirements are in conflict.

Cells which are not cylinders, but frustra of cones or ogives, and adjacent cells of unequal length, do not differ in principle but pose some complications in arithmetic.

Bulging of a bulkhead is affected by its structure, elastic properties, manner of support, and initial tension. Early rigid

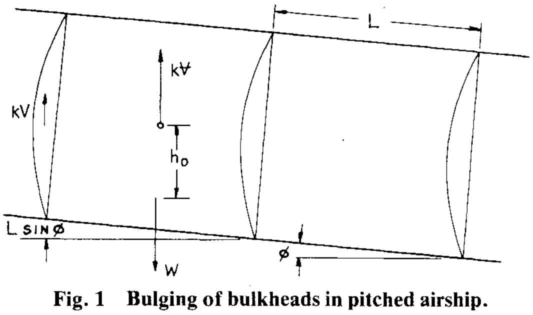

Fig. 1 Bulging of bulkheads in pitched airship.

Presented as Paper 81-1328 at the AIAA Lighter-than-Air Conference, Annapolis, Md., July 8-10, 1981; submitted July 21, 1981; revision received Dec. 11, 1981.

*President. Member AIAA.

airships had complex arrangements of chord wires as bulkheads, but the structures in more recent airships have primarily included two kinds: radial wires as used in the L.S. Hindenburg and post-U.S.S. Macon U.S. Navy proposals (Fig. 2), and "spiral netting" as in the U.S.S. Akron and Macon, various other Goodyear designs, and recently proposed large metalclads (Fig. 3). Elastic properties depend on the modulus of elasticity of the materials used and the cross-section areas required to carry the pressure stresses. The manner of support includes the use, or not, of a central restraint (an axial cable or girder) and whether the bulkhead anchorages are rigidly fixed to the frame or kept under constant or elastic tension, as by the pressure cylinders of the U.S.S. Akron and Macon. The initial tension is an independent design variable.

Complete treatments of these bulkhead design considerations have not appeared in the engineering literature. Burgess[1] covers the fundamentals of wires under initial tension and transverse loading, an essential element of bulkhead analysis, and he, as well as Lewitt,[2] also analyze sufficiently the loading of a frame by a radial or chordal wire bulkhead in the deflated-cell condition. Arnstein and Klemperer[3] give without derivation a formula for the stability effect of a radial wire bulkhead without axial restraint. Stability effects for the case of axial restraint, or any treatment of netting bulkheads, do not appear to have been published.

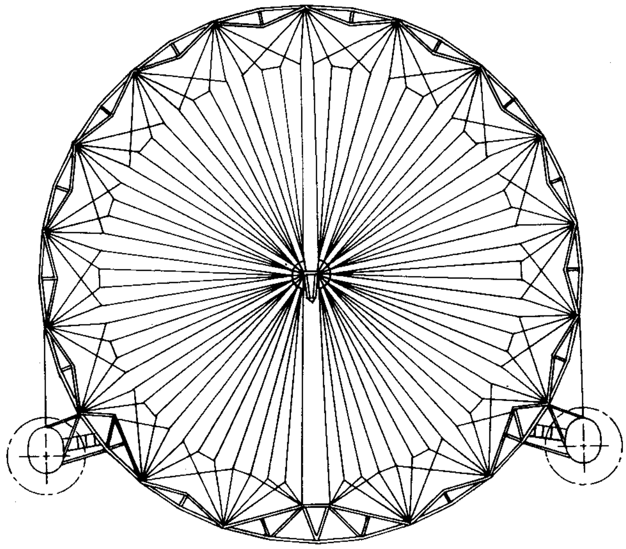

Fig. 2 Radial wire bulkhead of L.S. Hindenburg class.

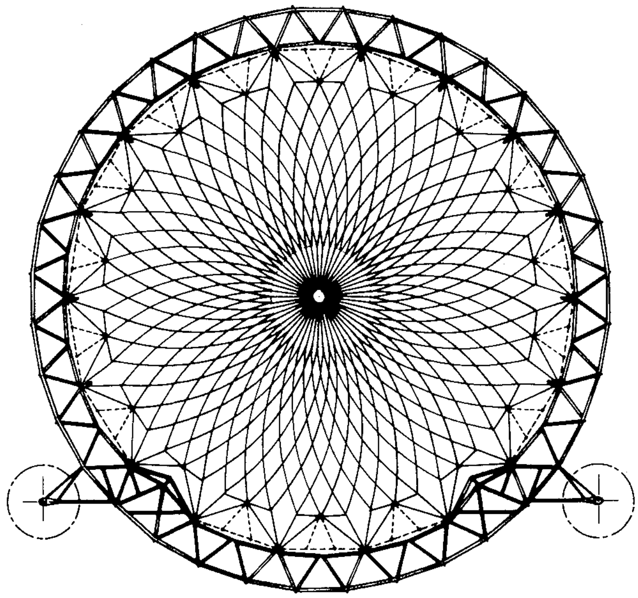

Fig. 3 Goodyear-Zeppelin netting bulkhead of U.S.S. Akron.

Radial Bulkheads

The case of a radial wire bulkhead under uniform pressure is essentially two dimensional. The equation of equilibrium of a short segment of wire with a distributed transverse load is

$$T=f\mathcal{R} \tag{1a}$$

which, when limited to small slopes (as throughout this paper), becomes

$$T\Big/\frac{d^2z}{dr^2}=f \tag{1b}$$

The triangular gore of bulkhead fabric that loads each wire increases in width proportional to r, so that

$$f=cr \tag{2}$$

The total transverse load on each wire is $P=\pi R^2p/n=cR^2/2$, and

$$c=2\pi p/n \tag{3}$$

Integrating twice,

$$Tz=\frac{1}{6}cr^3+C_1r+C_2 \tag{4}$$

For the case of no center restraint on the bulkhead, $z'(0)=0$ and $C_1=0$. $z(R)=0$, and therefore $C_2=-cR^3/6$ and

$$z=c(r^3-R^3)/6T \tag{5a}$$

which can be nondimensionalized as

$$Z=Y^3-1 \tag{5b}$$

Equation (5b) is plotted in Fig. 4.

Equations (5) are small-slope versions of the Taylor parachute profile.[4] The latter can be integrated in combinations of elliptic integrals, as could the bulkhead profile without the small-slope limitation. However, the boundary conditions are more difficult than for the parachute, for which θ inherently goes to 90 deg at the maximum diameter, and therefore $T=P$.

The difference between R and the arc length of the curve defined by Eq. (5a) is

$$(c^2/40T^2)R^5=RP^2/10T^2$$

Dividing by the original length R and equating to the change in wire strain,

$$T^3-T^2T_0-EaP^2/10=0 \tag{6}$$

An equation of this form holds for any transversely loaded wire, with the coefficient of EaP^2 dependent on the load distribution and end conditions.

The volume under the surface defined by rotating Eq. (5a) about the z axis is

$$V=\pi cR^5/10T \tag{7}$$

which is 3/5 the product of the maximum ordinate and the area of the base. Inserting $p=kL\sin\phi$ into Eq. (3), the

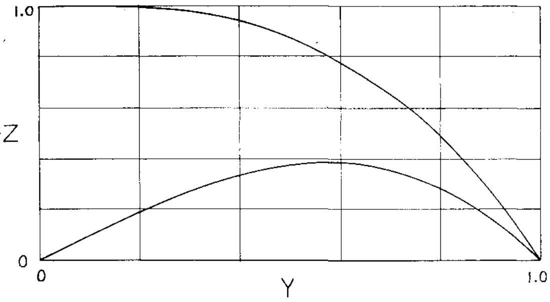

Fig. 4 Nondimensional bulkhead curves, $Z=Z(Y)$, from Eq. (5b) (upper curve) and Eq. (10b) (lower curve).

destabilizing moment will be

$$kVL=\frac{2k^2L^2\sin\phi R^5}{nT}=\frac{2k^2L\sin\phi R^3}{\pi nT}V \tag{8}$$

If the cell lift kV, is balanced by an equal weight at a distance h_0 below the center of volume, there will be a restoring moment $h_0\sin\phi kV$, less the destabilizing moment of Eq. (8). Defining the difference of moments as $(h_0-h_l)\sin\phi kV$, the reduction in metacentric height is given by

$$h_l=2kLR^3/\pi nT \tag{9}$$

(Arnstein and Klemperer[3] give a relation of the same form but with a coefficient of $5\pi/24$, which is 2.7% larger than $2/\pi$. Since they give no derivation, this difference cannot be explained.)

Equation (9) is valid in the limit $\phi=0$, giving $T=T_0$. If any value of h_l is chosen, the required pretension T_0 can be determined. The relation is equally valid when $\phi\neq 0$, when $P(=\pi kL\sin\phi R^2/n)$ will cause T to increase from T_0 according to Eq. (6), so that h_l will decrease as will the metacentric instability caused by gas surging. Even though for $T_0=0$, $h_l=\infty$ (i.e., complete instability), stability will be regained at some small pitch angle.

If the center of the bulkhead is fixed by an axial cable or girder, i.e., $z(0)=0$, the constants of integration in Eq. (4) change so that, instead of Eq. (5),

$$z=(c/6T)(r^3-rR^2) \tag{10a}$$

$$Z=Y^3-Y \tag{10b}$$

Equation (10b) is also plotted in Fig. 4. (These curves are also small-slope equivalents of generalized Taylor profiles discussed by Lester.[4]) The volume of the bulge is 4/9 that of the unrestrained bulkhead, and Eq. (9) becomes

$$h_l=8kLR^3/9\pi nT \tag{11}$$

$Z'(0)=-1$ and $Z'(1)=2$ so the axial support carries a third of the total pressure load on the bulkhead. In this case, Eq. (6) becomes

$$T^3-T^2T_0-2EaP^2/45=0 \tag{12}$$

In the foregoing cases of surging of gas in a fully inflated airship, all the wires have equal loads increasing linearly from zero at the center to the same maximum value at R. In the case of a deflated cell, this will be true only for horizontal wires, since the pressure across the bulkhead varies linearly with height above the cell bottom. Lewitt[5] has shown that, assuming the center of the bulkhead rigidly fixed, the top vertical wire would have five times as much total lateral load as the bottom wire; wires at intermediate angles would have intermediate loads. However, the center of the bulkhead, even with an axial restraint, will readily displace under the unbalanced wire tensions. Lewitt further showed that with only a small rise of the center the tensions will come into equilibrium at values which are nowhere more than 15% greater than those given by the uniform pressure relations, Eqs. (6) or (12).

The transverse frame will compress under the action of the bulkhead tensions, an effect which is equivalent to a reduction in E by an amount which can be calculated from the elastic properties of the frame girders. It is often of considerable benefit in reducing the bulkhead tensions. For the present purpose of comparing different bulkhead constructions, it may be ignored.

The great value of axial restraint, which was introduced by Johann Schütte in 1911, will be clear from a comparison of Eqs. (11) and (12) with (9) and (6). Consider an example based on the L.S. Hindenburg. The parameters are $R=20.3$ m, $L=16.5$ m, $k=1.1$ kg/m^3, $n=72$, $E=21{,}000$ kg/mm^2, $a=19.6$ mm^2 (wire diameter $=5$ mm), and $s=75$ kg/mm^2. The overall metacentric height h_0 of rigid airships has typically been on the order of $0.7R$; suppose that the allowable value of h_l is $\frac{1}{2}h_0=0.35R$, leaving the other half of h_0 to overcome the effects of sloshing of the cell bottoms[3] and aerodynamic pitching moments.

From Eq. (11), T_0 will be 84 kg. In the deflated-cell case, $P=\pi kR^3/n=400$ kg lateral load per wire. Equation (12) becomes

$$T^3-84T^2-2931\times 10^6=0$$

of which the solution is $T=1460$ kg. This corresponds to a tensile stress of 74.4 kg/mm^2, just below the allowable. There are four wires bridled to each of the 18 main joints of the frame, so each joint will have a radial load of 5840 kg. The compression in the transverse girders $=n'/2\pi$ (radial load), which amounts to 16,730 kg.

Consider next an airship with the same parameters but without axial restraint. From Eq. (9) the required value of T_0 is 190 kg. T for the deflated cell is found from Eq. (6):

$$T^3-190T^2-6596\times 10^6=0$$

yielding $T=1941$ kg. This tension corresponds to a stress of nearly 100 kg/mm^2, well above the allowable, so that a greater wire area is needed. To determine this, Eq. (6) can be divided through by a^3 to form a quadratic in a with s as parameter:

$$s^3a^2-s^2aT_0-EP^2/10=0 \tag{13}$$

$$421{,}875a^2-1{,}068{,}750a-336\times 10^6=0$$

of which the positive root is $a=29.5$ mm^2, for a wire diameter of 6.1 mm. The tension would be 2212 kg, 52% more than in the actual case with an axial girder, with a proportionate increase in frame weight.

In LZ-131, in order to decrease frame weight, the bulkhead wiring was special steel-wire rope with an elastic modulus of 15,000 kg/mm^2 and allowable tensile stress of 120 kg/mm^2. The geometry was the same as that of the L.S. Hindenburg except that the largest cells were lengthened to 18 m, which raised the required value of T_0 to 92 kg. Equation (12) in stress form becomes

$$s^3a^2-s^2aT_0-(2/45)EP^2=0 \tag{14}$$

The solution is $a=8.25$ mm^2, for which $T=990$ kg, or only 68% of the Hindenburg value. Dürr[6] gives the reduction in tension as 26½%; this may be because a wire rope does not develop the full strength and stiffness corresponding to the properties of its individual wires.

The axial support will deflect along the axis under the bulkhead load. Although it is easy to see that the volume of the bulge increases by that of a cone of base πR^2 and height X, this only acts to diminish the rate at which stability increases with ϕ, and has no effect on the determination of T_0 by Eq. (11). The deflection does increase T, however; the new tension may be calculated by adding to T_0 in Eq. (12) a term $EaX^2/2R^2$, which is the tension in an initially just-taut wire of length $2R$ that has its center displaced X by an applied load.

Continuing the LZ-131 example, if the axial cable had the same properties as the wire ropes in the bulkheads, and if the length from its anchorage at one end of the ship to the bulkhead in question were half the ship's length, or 131.5 m, then the displacement X under one-third the deflated-cell load of 28,900 kg would be 0.75 m. For the same working stress, a greater wire area would be required and Eq. (13) becomes

$$(s^3 - s^2X^2E/2R^2)a^2 - s^2T_0a - (2/45)EP^2 = 0 \qquad (15)$$

which gives $a = 8.6$ mm^2 and $T = 1033$ kg, i.e., a reduction of 29% of the Hindenburg value.

No consideration has yet been given to the role of the gasbag fabric itself in supporting part of the unbalanced bulkhead pressure. The extensional stiffness of the cell fabric is very much lower than that of the bulkhead wires, although its sectional area at the circumference is much greater. On the principle that stiffness attracts load, it might be supposed that the wiring would take by far the greater part of the load. Lewitt[7] has reported experiments on both small water models and full-size radially wired airships, in which the force on the axial cable was one-fourth the total bulkhead load. Measurements on the small models led him to conclude (mistakenly) that the wire curves were arcs of circles, in which case the longitudinal component of tension would be equal at both ends. Therefore the wires were thought to carry half the total load, and the fabric the other half. However, it has been shown above that the slope of the wire at the circumference is actually twice that at the axis; therefore the wires in Lewitt's tests would have supported three-quarters of the load and the fabric one-quarter. While the stiffness of the radial wires increases rapidly with the airship diameter, the extensional stiffness of the fabric (at least historically) does not, so that in the larger sizes now considered suitable for rigid construction the proportion of load taken by the fabric should be less. There will always be some relief of the bulkhead wiring by the fabric, however, with a small and imprecisely known but real increase in margin of safety. Full-scale bulkhead tests would determine this effect; Ref. 6, for example, has a photograph of such a test on a Hindenburg-type main frame, using an inflated air cushion to stress the bulkhead.

Netting Bulkheads

The U.S.S. Akron and Macon had netting bulkheads (Fig. 3). Similar bulkheads were proposed by Goodyear for later, unbuilt, rigid airships and have recently been shown in a series of design studies for large metalclad airship hulls.[8] Although usually described as spiral nets, in the Akron/Macon the individual wires zig-zagged from circumference to center, being fastened at their apparent crossings. If the net and the fabric it supports can be considered a quasimembrane, it is clear from the figure that it is both anisotropic and inhomogeneous. The rhombic meshes are unable, or nearly so, to transmit shear loads, which is unimportant for axially symmetric uniform pressure loads. For this case there are no shears and the surface is an equal-stress membrane. The principal stresses at any point are equal, all directions are principal directions, and all points are umbilical points.

For an isotropic and homogeneous membrane (and a net layout could be made that would come much closer to this than the Akron net), Trostel[9] gives an approximate large-deflection solution by Ritz's method, for $S_0 = 0$, that indicates that the shape of the meridian is not independent of the load intensity. In the limit of vanishing load, the shape approaches a parabola. Equating the product of edge tension S and $z'(R)$ to the total pressure load,

$$z = pR^2(1 - Y^2)/4S \qquad (16)$$

The volume under a paraboloid being half the height times the base area,

$$V = \pi pR^4/8S \qquad (17)$$

and, following the argument used to derive Eqs. (9) and (11),

$$h_l = kR^2L/8S \qquad (18)$$

from which S_0 can be obtained when a value is assigned to h_l. The pretension load at each of the frame joints will be $2\pi RS_0/n'$.

Because the function of the net is to transfer the bulkhead pressure load into the frame, the likely departures from isotropicity will be in the direction of increasing the ratio of radial to circumferential stiffness; clearly this is so for the Akron. Therefore it is safe to assume that the total preload on the bulkhead for given h_l will lie between that given by Eq. (9) and a value $\pi^2/8$ (≈ 1.23) times larger.

There obviously must be an equal-stress membrane surface with the center restrained, which could be approximated following Trostel. Equations (10) should be a good first approximation if the membrane is isotropic and homogeneous, and better if it is anisotropic. Since the inner part of the surface is anticlastic and the outer synclastic, it must be truly plane at z_{max}, where the stresses will be infinite. This is also true of the crown of the Taylor parachute, considered as a membrane rather than a rigging-line curve. The parachute does not burst, of course, and neither would the bulkhead. Although the deformations which prevent infinite stresses are obscure in both cases, they presumably have the effect of making the membrane anisotropic and inhomogeneous, which the spiral net already is. The Akron net is most anisotropic at the center, where otherwise it would approach the condition of the flat-crowned parachute. For a net bulkhead with axial support, it would be advisable to maintain high anisotropicity out to the region of z_{max}, which is at $Y = 1/\sqrt{3}$ for the shape given by Eq. (10b).

Two major problems remain. The first is to find the shape of the bulkhead, and from it the stresses and the loads on the frame, in the deflated-cell condition, with or without axial support. The other is to determine the effect on these quantities of the pretension S_0. Relaxation methods offer a possible approach; Shaw and Perrone[10] have treated the case of a rectangular membrane uniformly loaded, and there seem to be no fundamental reasons which prevent a similar solution with a circular boundary and nonuniform load. An alternate method is to consider the equilibrium of the net alone, assuming the fabric to be frictionless and incapable of supporting shear. This approach, which has been treated by Schleyer,[11] requires the solution of very large numbers of simultaneous equations.

The combination of asymmetric load and negligible shear stiffness will result in wrinkling. To the extent that the wrinkles are radially oriented, the membrane will approach the condition of the radial wire bulkhead, which is good. Because of the zig-zagging of the wires, actual wrinkles would not occur in a spiral net; instead, the vanishing circumferential stresses will result in slacking-off some of the wires, with major decreases in their apparent radial stiffness and major increases in their bulging. This is also good. The computational difficulties created by wrinkling, unless it can be shown to convert the membrane into an approximation of a radial wire bulkhead, will undoubtedly be severe.

Unfortunately, the only airships with net bulkheads, the U.S.S. Akron and Macon, provide no experimental results for guidance to the form of solution. In those airships the bulkhead stresses were kept low by the use of gas-pressurized "resiliency devices" at the anchorages to the frames, to permit large displacements with only small increases in load. This was feasible because the main frames of the hull were triangulated space frames that required no stiff transverse bracing for strength or structural stability. The bulkhead functions were thus preservation of the ships' longitudinal metacentric stability, and compartmentation for safety against general deflation.

Conclusions

The bulkhead and frame loads for the radial wire case with a deflated cell were long ago described by two classic authorities on airship design, Burgess and Lewitt. The influence on these loads of the wire pretensions necessary to secure a desired degree of metacentric stability in pitch has not previously been discussed quantitatively. The results derived in this study will help fill this gap. The examples given of the L.S. Hindenburg and LZ-131 bulkhead forces are proportionately in good accord with those given by their designer, Dr. Dürr. The incomplete, approximate results given for netting bulkheads are new. No experimental results that bear on their accuracy have been discovered. Although small-scale water-model tests have sometimes been used to verify bulkhead designs, confidence in them is reduced by an uncertain division of load between fabric and wires. Tests on full-size main frames, although obviously expensive, were preferred by Luftschiffbau Zeppelin. In addition, a structural proof-test by a gas cell inflated between empty bays has long been a usual requirement for rigid airships.

References

[1] Burgess, C.P., *Airship Design,* Ronald Press, New York, 1927, Chap. VIII.

[2] Lewitt, E.H., *The Rigid Airship,* Pitman, London, 1925, Chap. VII.

[3] Arnstein, K. and Klemperer, W., "Performance of Airships," in *Aerodynamic Theory,* edited by Durand, W.F., Vol. VI, Div. R. Springer, Berlin, 1936, pp. 58-60.

[4] Lester, W.G.S., "A Note on the Generalization of Elastic Curves Representing Parachute Shapes," Aeronautical Research Council, London, C.P. 665, 1963.

[5] Lewitt, E.H., *The Rigid Airship,* Pitman, London, 1925, pp. 134-138.

[6] Dürr, L.F., "Measures for Improving the Zeppelin Airships for Long Distance Transportation," NASA TM-76142, April 1980 (translated from *Schriften der Deutschen Akademie der Luftfahrtforschung,* Heft 2, 1938).

[7] Lewitt, E.H., *The Rigid Airship,* Pitman, London, 1925, pp. 130-133, 144-145.

[8] Campbell, L.R., Pavlecka, V.H., Roda, J.W., Stephens, E.V. and Szuladzinski, G., "Metalclad Airship Hull Study," Turbomachines, Inc., Irvine, Calif., Rept. NADC-76238-30 Vol. II, Dec. 1976.

[9] Trostel, R., "Calculation of Membranes," in *Tensile Structures,* edited by Otto, F., Vol. I, MIT Press, Cambridge and London, 1973, pp. 282-287.

[10] Shaw, F.S. and Perrone, N., "A Numerical Solution for the Non-Linear Deflection of Membranes," *Journal of Applied Physics,* Vol. 21, 1954, pp. 117-128.

[11] Schleyer, F.-K., "Analysis of Cables, Cable Nets and Cable Structures," in *Tensile Structures,* edited by Otto, F., Vol. II, MIT Press, Cambridge and London, 1973, pp. 99-145.